

8-4

Angles of Elevation and Depression

Common Core State Standards

G-SRT.C.8 Use trigonometric ratios . . . to solve right triangles in applied problems.

MP 1, MP 3, MP 4, MP 6

Objective To use angles of elevation and depression to solve problems



Did you know you could use geometry in theater? You can find math anywhere you look . . . up or down.



SOLVE IT!

Getting Ready!

You are on the lighting crew for the school musical. You hang a set of lights 25 ft above the stage. For one song, the female lead is on stage alone and you want all the lights on her. If she stands in the middle of the stage as shown, at what angle from horizontal should you set lamps A and B? Round to the nearest degree. Describe how each angle changes if you set the lamps for her to stand a few feet closer to the tree. (Diagram is not to scale.)



Lesson Vocabulary

- angle of elevation
- angle of depression

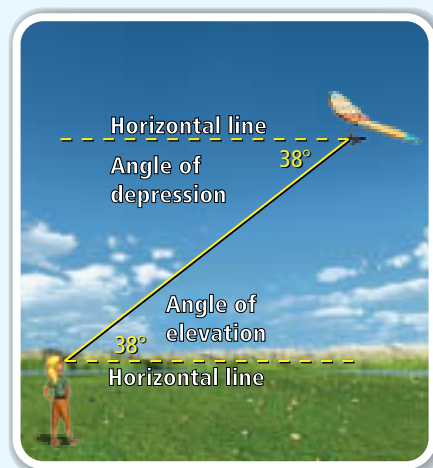
The angles in the Solve It are formed below the horizontal black pipe. Angles formed above and below a horizontal line have specific names.

Suppose a person on the ground sees a hang glider at a 38° angle above a horizontal line. This angle is the **angle of elevation**.

At the same time, a person in the hang glider sees the person on the ground at a 38° angle below a horizontal line. This angle is the **angle of depression**.

Notice that the angle of elevation is congruent to the angle of depression because they are alternate interior angles.

Essential Understanding You can use the angles of elevation and depression as the acute angles of right triangles formed by a horizontal distance and a vertical height.



Plan

How can you tell if it is an angle of elevation or depression?

Place your finger on the vertex of the angle. Trace along the nonhorizontal side of the angle. See if your finger is above (elevation) or below (depression) the vertex.



Problem 1 Identifying Angles of Elevation and Depression

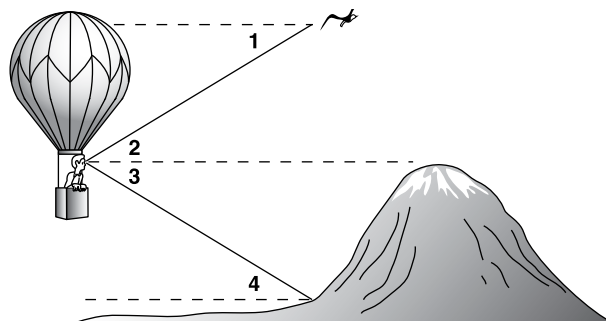
What is a description of the angle as it relates to the situation shown?

A $\angle 1$

$\angle 1$ is the angle of depression from the bird to the person in the hot-air balloon.

B $\angle 4$

$\angle 4$ is the angle of elevation from the base of the mountain to the person in the hot-air balloon.



Got It? 1. Use the diagram in Problem 1. What is a description of the angle as it relates to the situation shown?

a. $\angle 2$

b. $\angle 3$



Problem 2 Using the Angle of Elevation

Wind Farm Suppose you stand 53 ft from a wind farm turbine. Your angle of elevation to the hub of the turbine is 56.5° . Your eye level is 5.5 ft above the ground. Approximately how tall is the turbine from the ground to its hub?

$$\tan 56.5^\circ = \frac{x}{53}$$

Use the tangent ratio.

$$x = 53(\tan 56.5^\circ)$$

Solve for x .

$$53 \tan 56.5 \text{ enter}$$

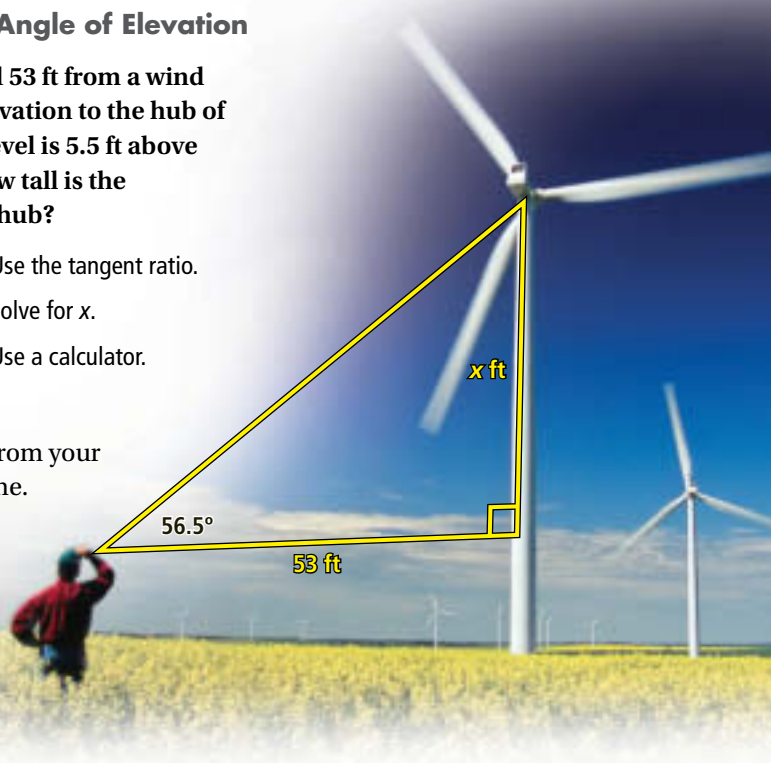
Use a calculator.

$$80.07426526$$

So $x \approx 80$, which is the height from your eye level to the hub of the turbine.

To find the total height of the turbine, add the height from the ground to your eyes.

Since $80 + 5.5 = 85.5$, the wind turbine is about 85.5 ft tall from the ground to its hub.



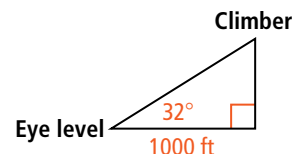
Think

Why does your eye level matter here?

Your normal line of sight is a horizontal line. The angle of elevation starts from this eye level, not from the ground.



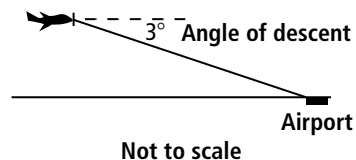
Got It? 2. You sight a rock climber on a cliff at a 32° angle of elevation. Your eye level is 6 ft above the ground and you are 1000 ft from the base of the cliff. What is the approximate height of the rock climber from the ground?



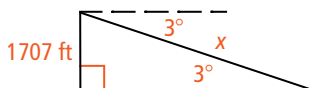


Problem 3 Using the Angle of Depression

To approach runway 17 of the Ponca City Municipal Airport in Oklahoma, the pilot must begin a 3° descent starting from a height of 2714 ft above sea level. The airport is 1007 ft above sea level. To the nearest tenth of a mile, how far from the runway is the airplane at the start of this approach?



The airplane is $2714 - 1007$, or 1707 ft, above the level of the airport.



$$\sin 3^\circ = \frac{1707}{x}$$

Use the sine ratio.

$$x = \frac{1707}{\sin 3^\circ}$$

Solve for x .

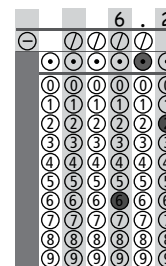
$$1707 \div \sin 3^\circ = 32616.19969$$

Use a calculator.

$$32616.19969 \div 5280 = 6.177310548$$

Divide by 5280 to convert feet to miles.

The airplane is about 6.2 mi from the runway.



Think

Why is the angle of elevation also 3° ?

The path of the airplane before descent is parallel to the ground. So the angles formed by the path of descent are congruent alternate interior angles.



- Got It?** 3. An airplane pilot sights a life raft at a 26° angle of depression. The airplane's altitude is 3 km. What is the airplane's horizontal distance d from the raft?

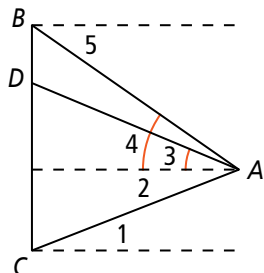


Lesson Check

Do you know HOW?

What is a description of each angle as it relates to the diagram?

- $\angle 1$
- $\angle 2$
- $\angle 3$
- $\angle 4$
- $\angle 5$

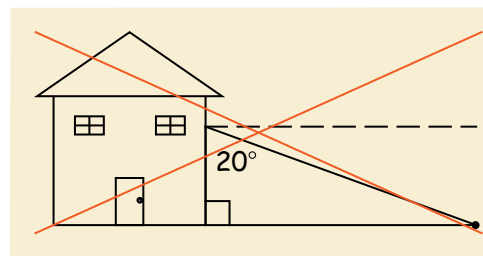


- What are two pairs of congruent angles in the diagram above? Explain why they are congruent.

Do you UNDERSTAND?



- Vocabulary** How is an angle of elevation formed?
- Error Analysis** A homework question says that the angle of depression from the bottom of a house window to a ball on the ground is 20° . Below is your friend's sketch of the situation. Describe your friend's error.





Practice and Problem-Solving Exercises



A Practice

Describe each angle as it relates to the situation in the diagram.

← See Problem 1.

9. $\angle 1$

10. $\angle 2$

11. $\angle 3$

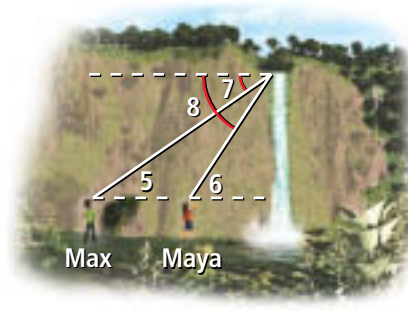
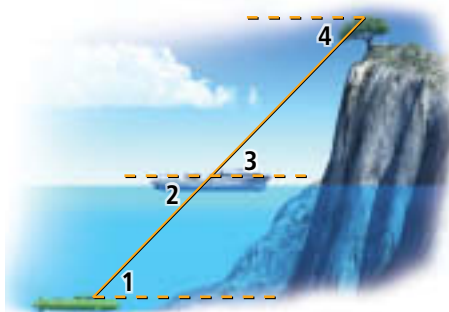
12. $\angle 4$

13. $\angle 5$

14. $\angle 6$

15. $\angle 7$

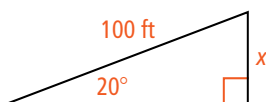
16. $\angle 8$



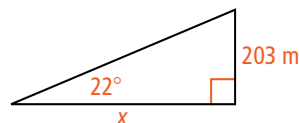
Find the value of x . Round to the nearest tenth of a unit.

← See Problem 2.

17.



18.

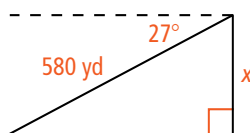


- STEM** 19. **Meteorology** A meteorologist measures the angle of elevation of a weather balloon as 41° . A radio signal from the balloon indicates that it is 1503 m from his location. To the nearest meter, how high above the ground is the balloon?

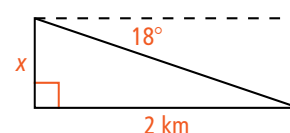
Find the value of x . Round to the nearest tenth of a unit.

← See Problem 3.

20.



21.



22. **Indirect Measurement** A tourist looks out from the crown of the Statue of Liberty, approximately 250 ft above ground. The tourist sees a ship coming into the harbor and measures the angle of depression as 18° . Find the distance from the base of the statue to the ship to the nearest foot.

B Apply

23. **Flagpole** The world's tallest unsupported flagpole is a 282-ft-tall steel pole in Surrey, British Columbia. The shortest shadow cast by the pole during the year is 137 ft long. To the nearest degree, what is the angle of elevation of the sun when casting the flagpole's shortest shadow?

- 24. Think About a Plan** Two office buildings are 51 m apart. The height of the taller building is 207 m. The angle of depression from the top of the taller building to the top of the shorter building is 15° . Find the height of the shorter building to the nearest meter.
- How can a diagram help you?
 - How does the angle of depression from the top of the taller building relate to the angle of elevation from the top of the shorter building?

Algebra The angle of elevation e from A to B and the angle of depression d from B to A are given. Find the measure of each angle.

25. $e: (7x - 5)^\circ$, $d: 4(x + 7)^\circ$

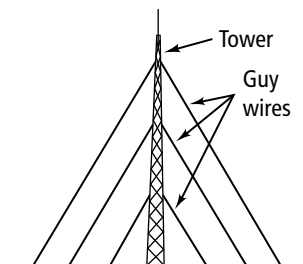
26. $e: (3x + 1)^\circ$, $d: 2(x + 8)^\circ$

27. $e: (x + 21)^\circ$, $d: 3(x + 3)^\circ$

28. $e: 5(x - 2)^\circ$, $d: (x + 14)^\circ$

- 29. Writing** A communications tower is located on a plot of flat land. The tower is supported by several guy wires. Assume that you are able to measure distances along the ground, as well as angles formed by the guy wires and the ground. Explain how you could estimate each of the following measurements.

- the length of any guy wire
- how high on the tower each wire is attached



Flying An airplane at a constant altitude a flies a horizontal distance d toward you at velocity v . You observe for time t and measure its angles of elevation $\angle E_1$ and $\angle E_2$ at the start and end of your observation. Find the missing information.

30. $a = \blacksquare$ mi, $v = 5$ mi/min, $t = 1$ min, $m\angle E_1 = 45$, $m\angle E_2 = 90$

31. $a = 2$ mi, $v = \blacksquare$ mi/min, $t = 15$ s, $m\angle E_1 = 40$, $m\angle E_2 = 50$

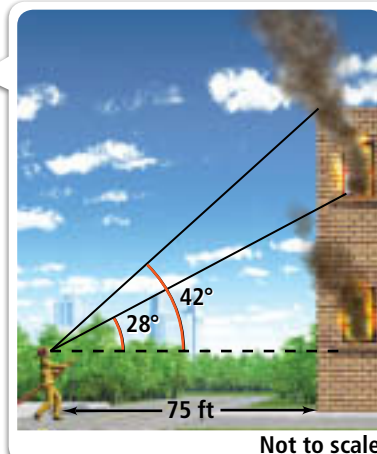
32. $a = 4$ mi, $d = 3$ mi, $v = 6$ mi/min, $t = \blacksquare$ min, $m\angle E_1 = 50$, $m\angle E_2 = \blacksquare$

- 33. Aerial Television** A blimp provides aerial television views of a football game. The television camera sights the stadium at a 7° angle of depression. The altitude of the blimp is 400 m. What is the line-of-sight distance from the television camera to the base of the stadium? Round to the nearest hundred meters.



Challenge

34. **Firefighting** A firefighter on the ground sees fire break through a window near the top of the building. The angle of elevation to the windowsill is 28° . The angle of elevation to the top of the building is 42° . The firefighter is 75 ft from the building and her eyes are 5 ft above the ground. What roof-to-windowsill distance can she report by radio to firefighters on the roof?
35. **Geography** For locations in the United States, the relationship between the latitude ℓ and the greatest angle of elevation a of the sun at noon on the first day of summer is $a = 90^\circ - \ell + 23.5^\circ$. Find the latitude of your town. Then determine the greatest angle of elevation of the sun for your town on the first day of summer.



Standardized Test Prep

SAT/ACT

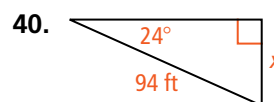
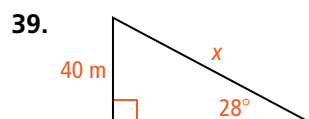
36. A 107-ft-tall building casts a shadow of 90 ft. To the nearest whole degree, what is the angle of elevation of the sun?
- (A) 33° (B) 40° (C) 50° (D) 57°
37. Which assumption should you make to prove indirectly that the sum of the measures of the angles of a parallelogram is 360° ?
- (F) The sum of the measures of the angles of a parallelogram is 360° .
 (G) The sum of the measures of the angles of a parallelogram is not 360° .
 (H) The sum of the measures of consecutive angles of a parallelogram is 180° .
 (I) The sum of the measures of the angles of a parallelogram is 180° .

Extended Response

38. A parallelogram has four congruent sides.
- Name the types of parallelograms that have this property.
 - What is the most precise name for the figure, based only on the given description? Explain.
 - Draw a diagram to show the categorization of parallelograms.

Mixed Review

Find the value of x . Round to the nearest tenth of a unit.



See Lesson 8-3.

Get Ready! To prepare for Lesson 8-5, do Exercises 42–44.

Find the distance between each pair of points.

See Lesson 1-7.

42. $(0, 0)$ and $(8, 2)$ 43. $(-15, -2)$ and $(0, 0)$ 44. $(-2, 12)$ and $(0, 0)$